AP Calculus AB – Summer Assignment

DUE: First day of school, 2019. Use graphing paper; show all your work neat and organized. You have to rewrite the given problem where you solve it, make sure you write the problem number.

Complete this assignment at your leisure during the summer. I strongly recommend you complete a little each week. The assignment is designed to help you become more comfortable with your graphing calculator, review basic algebra skills, review basic trigonometry skills, and review families of functions. It is important for you to review or gain these skills during the summer so that we can spend our time learning Calculus.

Families of Functions. YOU DO NOT HAVE TO GRAPH THESE BY HAND!!!
GOAL: You should be able to recognize each type of parent (or basic) function. You should be able to identify the domain and range as well as any special characteristics of each function based on its "family" and its transformations.
Using your graphing calculator or a graphing program such as Desmos, re-familiarize yourself with the basic functions and their transformations. Graph each parent function. One by one, on the same window as the parent function, graph each transformed function. Be able to describe the transformations caused by each constant.

1) $\boldsymbol{y = x^2}$

a) $y = x^2 - 5$ b) $y = x^2 + 3$ c) $y = x^2 + 0.23$ d) $y = x^2 - 0.34$

e) $y = (x - 5)^2$ f) $y = (x + 3)^2$ g) $y = \left(x + \frac{1}{2}\right)^2$ h) $y = \left(x - \frac{1}{3}\right)^2$

i) $y = 4x^2$ j) $y = 0.25x^2$ k) $y = -x^2$ l) $y = -3.5x^2$

QUESTION: (Answer in a full sentence.) Given $y = a(x - h)^2 + k$, define how $a, h,$ and k transform the function.

2) $\boldsymbol{y = x^3}$

a) $y = x^3 - 5$ b) $y = x^3 + 3$ c) $y = x^3 + 0.23$ d) $y = x^3 - 0.34$

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i) $y = 4x^3$ j) $y = 0.25x^3$ k) $y = -x^3$ l) $y = -3.5x^3$

QUESTION: (Answer in a full sentence.) Given $y = a(x - h)^3 + k$, define how $a, h,$ and k transform the function.

3) $\boldsymbol{y = \sin(x)}$

a) $y = \sin(x - 4)$ b) $y = \sin\left(x + \frac{\pi}{2}\right)$ c) $y = \sin(x) + 5$

d) $y = \sin(x) - 6$ e) $y = \sin(2x)$ f) $y = \sin\left(\frac{1}{4}x\right)$

g) $y = 2\sin(x)$ h) $y = \frac{1}{4}\sin(x)$ i) $y = -3\sin(x)$

j) $y = \sin(2x - 8)$ *How is this equation similar to two other equations above?

k) $y = \sin\left(-3x + \frac{\pi}{2}\right) + 6$

QUESTION: (Answer in a full sentence.) Given $y = a\sin\big(b(x - h)\big) + k$, define how $a, b, h,$ and k transform the function.

4) Repeat question 3 using cos (x) instead of sin (x).

5) $y = \sqrt{x} = x^{1/2}$

a) $y = 4\sqrt{x}$ b) $y = -3\sqrt{x}$ c) $y = \frac{1}{2}\sqrt{x}$ d) $y = -\frac{1}{3}\sqrt{x}$

e) $y = \sqrt{4x}$ f) $y = \sqrt{9x}$ g) $y = \sqrt{x-4}$ h) $y = \sqrt{x+6}$

i) $y = \sqrt{x} + 7$ j) $y = \sqrt{x} - 3$

QUESTION: (Answer in a full sentence.) Given $y = a\sqrt{b(x-h)} + k$, define how $a, b, h,$ and k transform the function.

6) $y = \log_{10} x = \log x$

a) $y = \log(x+3)$ b) $y = \log(x-5)$ c) $y = \log x + 7$

d) $y = \log x - 9$ e) $y = 3\log x$ f) $y = \frac{1}{2}\log x$

g) $y = -4\log x$ h) $y = \log_4 x$ i) $y = \log_5 x$

QUESTION: (Answer in a full sentence.) Given $y = a\log_b(x-h) + k$, define how $a, b, h,$ and k transform the function. Why can b NOT be a negative value?

7) $y = e^x$

a) $y = 4e^x$ b) $y = -3e^x$ c) $y = e^{x-4}$ d) $y = e^{x+8}$

e) $y = e^x + 4$ f) $y = e^x - 3$ g) $y = \frac{1}{2}e^x$

QUESTION: (Answer in a full sentence.) Given $y = ae^{x-h} + k$, define how $a, h,$ and k transform the function.

8) $y = b^x$

a) $y = 2^x$ b) $y = 3^x$ c) $y = \left(\frac{1}{2}\right)^x$ d) $y = 3(2^x)$

e) $y = 2^{x-4}$ f) $y = 3^{x+5}$ g) $y = 2^x + 5$ h) $y = 3^x - 7$

QUESTION: (Answer in a full sentence.) Given $y = ab^{x-h} + k$, define how $a, h,$ and k transform the function.

9) $y = \frac{1}{x}$

a) $y = \frac{1}{x-2}$ b) $y = \frac{1}{x+4}$ c) $y = \frac{1}{x} + 6$

d) $y = \frac{1}{x} - 4$ e) $y = \frac{5}{x}$ f) $y = -\frac{3}{x}$

QUESTION: (Answer in a full sentence.) Given $y = \frac{a}{x-h} + k$, define how $a, h,$ and k transform the function. Explain *how* to identify the vertical asymptote. Then explain *why* your process works.

<u>Trigonometry</u>

GOAL: To know the unit circle by heart, be able to remember the unit circle when a calculator is not allowed to evaluate trigonometric functions, and to solve trigonometric equations with or without a calculator.

10) Recall that:

$\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$ and $\cot x = \frac{1}{\tan x}$

Use these trigonometric identities to review and become familiar with the graphs of $\sec x$, $\csc x$, and $\cot x$.

QUESTION: Why can't we graph $\sec x$ by graphing $\cos^{-1} x$? Your answer should include a clear differentiation of a reciprocal and an inverse function.

11) MEMORIZE the unit circle. Be prepared to explain the unit circle and to use the unit circle to find the sine, cosine, tangent, secant, cosecant, and cosecant of the angles $\pi, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$ and their multiples *without* a calculator.

12) MEMORIZE basic trig identities. Know the reciprocal, quotient, and all Pythagorean identities.

a) Use the fact that $\mathbf{\sin^2 x + \cos^2 x = 1}$ to prove that $\mathbf{\tan^2 x + 1 = \sec^2 x}$
b) Use trig identities, the unit circle, and algebra to solve trig equations like:
$\mathbf{10\sin^2 x - 9\cos x - 12 = 0}$

<u>**Basic Algebra Skills**</u>
GOAL: To develop facility with these skills so that (with or without a calculator) you can spend your time thinking about calculus, not algebra!!

13) Simplify or rationalize as needed:

a) $\frac{3x}{x+1} - \frac{4}{x^2-1}$ b) $\frac{2x+3}{x-2} + \frac{3x}{x+2}$ c) $\frac{2}{x+3} + \frac{x}{x^2+5x+6} - \frac{2x}{x+2}$ d) $\frac{\sin\theta}{5\cos\theta} + \frac{\cos\theta}{5\sin\theta}$

e) $\frac{15x}{\sqrt{5-\cos x}}$ f) $\frac{13a}{\sqrt{5-a}}$ g) $\frac{1}{\sqrt{7}}$

14) Simplify:

a) x^4x^{15} b) $(x^3)^4$ c) $3(x^4)^8$ d) $(3x^4)^5$

e) $\sqrt[5]{64x^{10}}$ f) $\sqrt{\sin^2 x + \cos^2 x}$

15) Solving equations is an **ESSENTIAL SKILL** in Calculus. Do not be fooled by the small number of examples here!!

Solve – **Using ALGEBRA** – do not just graph and find the intersections:

a) $144 = 5x^2 - 8x + 11$ b) $0 = 9x^5 + 72x^2 + 4x^4 + 32x$ c) $450 = x^2 - 35x$

d) $0 = 5x^3 + 45x^2 - 20x$ e) $15 = 10(3^{4x-10}) - 25$ f) $-4\ln(5x+2) = 11$

g) $0 = e^x(x^2 - 1)$ h) $0 = (x-2)(x-3)$ i) $x^2 - 5x + 6$

j) $15x^2 + x - 2 = 0$ k) $0 = e^x x^2 - e^x$

16) For each set of given information, write the equation that represents the situation:
a) $\boldsymbol{p}$ is directly proportional to $\boldsymbol{z}$. When $\mathbf{p = 7, z = 21}$.
b) $\boldsymbol{t}$ is inversely proportional to $\boldsymbol{n}$. When $\mathbf{t = 45, n = 3}$.
c) A population of bears doubles every 20 years. In year 5, the population was 550 bears.
d) The diagonal of a square can be found using the expression $3x^2 + 5$. What is the area of the square?
e) A line that goes through the points $(5, 7)$ and $(19, -45)$.
f) A line with a slope of $\frac{4}{5}$ that goes through the point $(2, 3.6)$.
g) A line perpendicular to $3x - 4y = 7$ that goes through the point $(21, 11.3)$.

17) Without a calculator, using rules of logs or exponents, determine if the first value is greater than, less than, or equal to the second value.

a) $\ln 49$ vs. $3\ln 7$
b) $\log_{132} 8$ vs. $\log_2 1024$
c) $e^{\ln 15}$ vs. $8^{4/3}$
a) $\ln(7/4)$ vs. $\ln(7/5)$

18) Write the explicit functions implicit in the equation – i.e.: solve for y:

$$3x^2 - 5y^2 = 21$$

19) Find the inverse function for $y = \frac{\sin(5x+7)-35}{12.5}$

20) If $\boldsymbol{f(x) = 2x^3 - 14}$ and $\boldsymbol{g(x) = 4\sqrt{3x+5}}$, evaluate:

a) $f(10)$ b) $g(10)$ c) $f\big(g(2)\big)$ d) $f^2(-3)$ e) $g^{-1}\big(g(12)\big)$

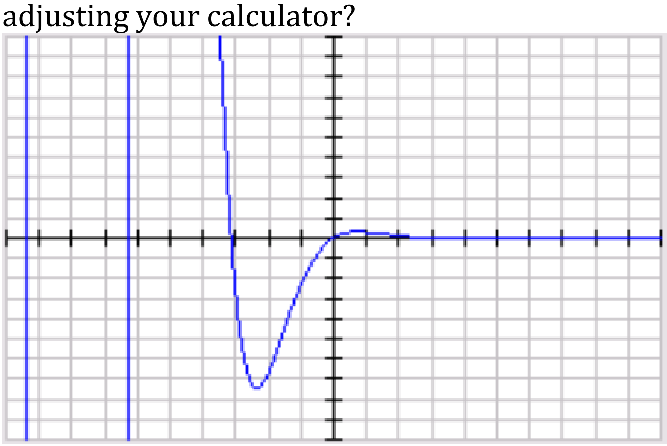
<u>Calculator Skills</u>

GOAL: To be able to use the tool to support your mathematical thinking.

You should be familiar with these skills If necessary, review your owner's manual to understand how to perform each function. You can find the one specific to your model in the following link: http://education.ti.com/en/us/guidebook/search/graphing-calculators

21) The graph of $y = \frac{\sin x}{e^x}$ is shown below on the window shown. Based on the window below, how many zeros does the function have on [0, 10]?

Explain why this calculator screen does not provide a good indication of the zeros on the interval. How many zeros should you have on the interval? How do you know? How could you "see" these zeros by adjusting your calculator?



22) Give step-by-step instructions for graphing

a) $y = |x|$ b) $y = [x]$ (The greatest integer function)

23. Solve by graphing. State all steps.

$$\tan^{-1} x = x^2 + 2x - 3$$

24. Enter the following functions in your "y=" window.

$f(x) = y_1 = \sin(x)$ $\qquad g(x) = y_2 = -3x^2 + 5$

Store the value 12.314 in A and the value -4.56 in B.

Now, using the VARS menu, evaluate the following on your home screen.

a) $f(A)$ $\qquad$ b) $g(4B)$ $\qquad$ c) $5f(20) - 3g(2B)$

25. Enter $h(x) = x^3 + 9$ into y_1.
Then, evaluate the following:
a) $y_1(2)$ $\qquad$ b) $y_1{}^{-1}(2)$ $\qquad$ c) $[y_1(2)]^{-1}$

26. Explain why the expression in 25 b and 25 c are NOT equivalent.

27. Explain why the expression in 25 c IS equivalent to 1/17.

28. Write the equation of the line through (2, -7) and (10, -3)
a) in slope-intercept form $[y = mx + b]$
b) in point-slope form $[(y - y_1) = m(x - x_1)]$
c) in standard form $[Ax + By = C]$

29. Make a geometry review sheet by reviewing or finding the formulas for:
Area: circle, rectangle, triangle (look for the simplified version for equilateral triangles), parallelogram, and trapezoid.
Surface Area: cylinder, sphere.
Volume: sphere, right pyramid, right circular cone, cylinder.

Remaining Questions:
In your notebook, record any questions you have about the basic skills you have reviewed. Be sure your questions are clear, refer to a specific question or skill and are detailed so that the discussion in class the first few days can be productive.

Key Skills List (embedded in the summer assignment)

Factoring	Finding inverse functions	Solving for a variable using the zero product rule, graphing, and the quadratic formula.
Distribution	Basic geometric formulas	Identification of and understanding of characteristics of basic functions
Arithmetic of fractions	Right triangle trigonometry	Transformations
Function notation	Writing equations of lines in all three forms	Basic geometric formulas
Composition of functions		